Could We Be Wrong? An Exploration into Mathematical Truths, First Principles, Where We Were Wrong, and How We Could Be Wrong Again.

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“We would be advised [above] all to pay heed to the way, and not to fix our attention on isolated sentences and topics” (Heidegger). Be mindful of the above quote when perusing the following thesis.

Preface

Could we be wrong? Simple answer: Yes. We have constructed erroneous hypotheses and theories in the past and cannot be utterly prevented from fabricating them in the future. Speaking specifically about mathematics, we could be mistaken about our fundamental axioms. So, what is truth? More concretely, what is mathematical truth? How could theories which satisfy either one of the two definitions of mathematical truth[1] lead to inaccurate conclusions? And what is the solution to and how do we cope with fallacious arguments? These are the question that will be addressed in the course of the subsequent thesis.

First, it is advantageous to understand the definition and the etymology of the word *true*. The following are different derivations from both Old English and Greek.

> “In several Old English dialects, the word *treow* was a noun that meant good faith or trust, a pledge or a promise. But it also had another definition, tree, and that's no coincidence. If we trace the roots back even farther, we find that both meanings derive from a common origin, where some of the earliest expressions of the concept of truth were associated with the uprightness of an oak, the steadiness of a silver birch, and the fidelity of an orchard bearing fruit year after year.” (Gina Cooke[2]).

[1] The two definitions of mathematical truth addressed here will be expounded upon later in the text.
[2] ed.ted.com

Facts, things which are considered true, are seen as indisputable statements not contingent upon subjectivity. Facts are either true or false, exist or imaginary. Trees (obviously) are true, existing entities which posses validity and whose existence cannot be contradicted. Truth is stable, reliable, and solid. The comparison with a tree is apt since trees are not flimsy; they are solid fixtures rooted strongly to the ground..

In contrast, the Greek word for *true* is *ἀλήθεια,* whose origins come from the word *λήθη*. *Λήθη*

> "was the underworld river of oblivion...The shades of the dead drank of its waters to forget their mortal lives." (Aaron J. Atsma[3])

The definition of *λήθη* is forgetting or forgetfulness (Liddell and Scott[4]). Thus *ἀλήθεια* means un-forgetting or un-forgetfulness since *ἀλήθεια is λήθη* with an alpha privative[5]. The concept of un-forgetting is explained through texts such as the *Meno* in the form of recollection, which is the Platonic theory of how humans re-attain *a priori* knowledge (or Forms) through remembrance. Thus truth, in this sense, are the Forms. Something that we have learned in another life since forgotten and when recollected it is branded as an *a priori* truth.

Recollection entails remembering Forms or features of the Forms[6]. But, as humans, we can never reason or cognize all the way to the Forms themselves as objects[7] therefore we recollect them. Examples of recollected Forms are concepts such as Time and Space. Such concepts are neither conclusions in themselves nor first principles. First principles (or common

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[3] theoi.com
[4] *Greek-English Lexicon*
[5] When an alpha privative is placed in front of a word, it causes the word to mean its opposite.
[6] The capitalization of a word signifies the form of the word i.e. Beauty is the form of beauty or Truth is the form of truth
[7] Forms themselves as objects inasmuch as Truth itself or Beauty itself. The Form in its original state without any other attributes.

notions) are either (1) ideas uncovered through logic or (2) seemingly obvious, uncontradictable concepts which are therefore assumed to be true. In either case, the Forms satisfy none of the requirement to be classified as first principles. You may be saying: The concepts found through recollection and these above mentioned assumed truth[8] sound an awful lot alike. Therefore, aren't they the same thing? Simply: No. Recollected Truths are concepts reality itself is hinged upon—Space, Time—or necessary concepts—Beauty, Justice. Our world could not exist without these Truths. We may hypothesize about them, but their existence itself cannot be questioned. These hypotheses regarding Forms can be and are scrutinized; their veracity is contested, but for the sake of ascertaining Truth. Recollection does not bring forth mathematical truth or theories but the existence of True concepts. When intellectuals theorize about these *a priori* concepts and the theories become accepted[9] as truth, they become common notions—gravity has become one such notion. We have taken these truths for granted and forget the remarkable process of their discovery. Recollection brings Forms, which differ from first principles in the above manner. So now having explained the etymology of the word *true*, the definition we will be working with in this paper should now be explained.

So what is mathematical truth? As alluded to before, there are two varieties of mathematical truth. There are (1) logical progressions and (2) mathematics which adhere to reality or empirical knowledge (for the sake of clarity, this definition can be referred to as empirical mathematics). The first realm can be replaced with geometry and the second with

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[8] Assumed truths denoting the later definition of first principles. Inasmuch as the first principles are not discovered through logic.

[9] The concepts are accepted not universally, but generally. A current example is the use of the metric system around the world. Only three countries around the world do not employ the system: the United States, Myanmar, and Liberia. Therefore the metric system is generally accepted but not universally.

physics. Logical consistency is when the internal system produces a conclusion that can be defined as truth. In other words, when the conclusion follows logically from the assumed first principles. I say you can replace this form with geometry since the form of geometry itself is a series of logical progression. The conclusions made in Euclid are logical deductions made from his axioms, definitions, and common notions[10]. In addition, the propositions are placed in a specific fashion so the next in the succession uses the former propositions as a means to prove the validity of the conclusion.

The second form of mathematical truth—empirical mathematics—can be explained as mathematical conclusions which, not only logically follow from their first principles, but also conform to our reality (our perception of the world). Galileo, Newton, Einstein, Ptolemy all have mathematical theories which correspond to the empirical world around them. The theories are not just pen and paper, but also calculations about motion, the moon and star, and our reality. Since the first principles have to be correct and the empirical evidence, discovering truth is a considerably more difficult task to accomplish in the second realm of mathematical truth.

The question now is: Does either definition attain Truth? According to both Socrates and Einstein, the first definition holds no real Truth value. First, in the talk of the divided line in Plato's *Republic,* Socrates explains how first principles are chosen without adequate testing or explanation:

> "[A mathematician] makes these their hypotheses and don't think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone. And going from these first principles through the remaining steps, they arrive in full agreement" (Plato, *Republic* 510 c/d).

[10] All of which can be considered first principles.

When first principles are assumed or not given explanation and the conclusion logically follows, Socrates asks why that would be real Truth? The conclusion achieved is solely a proof showing the argument is internally faultless[11]. The audience acquires no knowledge about reality or anything greater about the world. Does the argument hold for phenomenons outside itself? Or is it just an interesting argument or theory which all fits together neatly? And it is not astonishing if the argument fits together when the mathematician picked the first principles to adhere to the conclusion or vice versa. In addition, not explaining or vetting first principles can also cause problems[12] since using them as a basis of a theory when they are incorrect can cause the whole theory to become obsolete. There is no real Truth according to Socrates achieved from the first definition of truth: logical progression or geometry.

Einstein is in agreement with the above assessment on mathematical truth[13]. Are logical progressions wrong? Technically, no. The theories themselves are not wrong, but correlation with reality is not the end goal. As Einstein states in his book *Relativity*:

> ...from certain simple propositions (axioms) which, in virtue of these ideas, we are inclined to accept as 'true'. Then, on the basis of a logical process, the justification of which we feel ourselves compelled to admit, all remaining propositions are shown to follow from these axioms, i.e. they are proven. A proposition is then correct ('true') when it has been derived in the recognized manner from the axioms. The question of the 'truth' of the individual geometrical propositions is thus reduced to one of the 'truth' of the axioms. Now it has long been known that the last question is not only unanswerable by

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[11] A logical argument progressing from an appointed starting point.

[12] This will be demonstrated later on the topic of Ptolemy and empirical mathematics.

[13] Sidenote: Now, even though Einstein and Socrates agree that geometry does not achieve truth, they diverge on how to attain truth. Socrates goes the route of the dialectic; searching for truth via conversation. Some may argue he never find truth. Every time the conversation seems on the verge of achieving some end, the interlocutors fall short of discovering truth. Einstein turns to physics for truth. Finding that truth is found in the second definition of mathematical truth—with empirical knowledge. Socrates goes the philosophy route while Einstein goes the physics one. Regardless of the path taken truth is not guaranteed. The following example is evidence that truth cannot be achieved through the first definition of mathematical truth.

> the methods of geometry, but that it is in itself entirely without meaning. We cannot ask whether it is true that only one straight line goes through two points. We can only say that Euclidean geometry deals with things called 'straight lines,' to each of which is ascribed the property of being uniquely determined by two points situated on it. The concept 'true' does not tally with the assertions of pure geometry, because by the word 'true' we are eventually in the habit of designating always the correspondence with a 'real' object; geometry, however, is not concerned with the relation of the ideas involved in it to objects of experience, but only with the logical connection of these ideas among themselves… the propositions of Euclidean geometry then resolve themselves into propositions on the possible relative position of practically rigid bodies… In less exact terms we can express this by saying that by 'truth' of a geometrical proposition in this sense we understand its validity for a construction with ruler and compasses. (*Relativity,* Einstein, 7-9).

More concisely, geometry is not concerned about corresponding with reality but about internal correctness. Therefore these logical progressions are not technically wrong; they are just not fruitful in discovering real Truth. They adhere to the first definition of mathematical truth but they do not satisfy the second definition. In addition, this type of logical progression is technically neither more true nor more false than any other type of logical progression that is only internally correct. This can be seen in the example to follow with Lobachevski and Euclid.

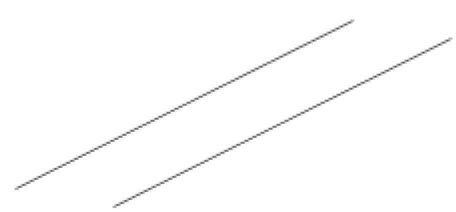
Euclid Verses Lobachevski

To reiterate, geometry falls under the first definition of mathematical truth and both a mathematician/physicist and a philosopher said no real Truth comes from this avenue. So, let us test this theory. Geometry is almost the most basic mathematical process at our disposal after arithmetic. If a fundamental theory in geometry can be changed or supplemented with a different idea, then was the original theory correct to begin with? And, does logical progression provide a solid, unquestionable truth for mathematics? In other words, does geometry provide us with any reassurance that we could have a constant in mathematics? Could we have found an unwavering Truth? Is there anything in mathematics which cannot be touched by doubt? If, yes, there is a constant which cannot be modified in mathematics, then the main question of the essay[14] will seem like a pointless endeavor. But, if the answer is no and basic mathematics cannot hold up under scrutiny, then the main question still stands. Could we be wrong? If Euclidean theories can be doubted, then almost any more complex theory be questioned as well?[15] So let us "get down to brass tacks" as the saying goes and look to Euclid.

I will now examine Euclid's theory of parallel lines, which he includes in the postulates and definitions causing them to appear as basic ideas. Then I will contrast Euclid with Lobachevski on this matter. Euclid leaves an opening in his explanation of parallel lines that permits Lobachevski to supplement his theory. But first let me examine what Euclid states on the matter of parallel lines. In Definition Twenty-three the reader receives their first encounter with parallel lines. It states the following:

> "Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction" (Euclid, 2).

The figure to the right gives a depiction of the definition above. At first glance, the concept seems obvious and



[14] The main question: Could we be wrong?
[15] This question comes from the idea simple theories are easier to prove. Theory: the sky is currently blue. Proof: observations of the sky at the given time show the theory to be correct. Therefore, if the simpler proof can be called into doubt, the more complex theory will be doubted as well.

without fault. Is this because we are previously acquainted with the definition? Does it seem like a common notion because it is inherently correct? I would argue the former. As Henri Poincaré says:

> “facts which occur frequently appear to us simple just because we are accustomed to them” (*Science and Method*, 19).

But within the definition above, there are concepts which cannot be counted as obvious. The essential concept which is called into question is the use of “indefinitely”. Meaning, if the lines were produced to infinity they would never touch. But, can this be proven? If yes, how? No, the concept can never be proven because we cannot produce an indefinite line in actuality. The concept must then be taken as theoretical causing the reader to be uncertain of the validity of the statement.

The other defining factor Euclid describes to parallel comes from Postulate Five. In his postulate it states:

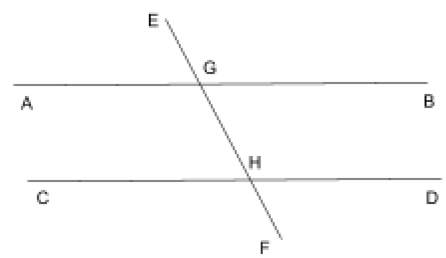
> “That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles” (Euclid, 2).

Essentially saying: If two parallel lines are cut by a third, the interior angles on the same side equal two right angles and, if not, the angles will be less than two right angles thus causing the lines to intersect. But the postulate is off the assumption a line can be produced indefinitely and there is only one line which does not intersect another line: a parallel line.

The first application of Postulate Five is in Book One Proposition Twenty-nine which states:

> "A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles." (22)

The figure to the right is exactly the same as the figure given by Euclid in his proof. Postulate Five is proven by way of reduction ad absurdum in which the postulate is the contradiction to one of the first fabricated, false premises.



Euclid takes two equal angles AGH and GHD and says that if they are unequal let AGH be bigger. Next, add the same angle (BGH) to each AGH and GHD. It will follow that AGH, BGH will be greater than GHD, BGH. From the properties known about a straight line and angles, we know that AGH and BGH added together will equal two right angle. Since it was said earlier that AGH, BGH will be greater than GHD, BGH, then GHD and BGH must add to less than two right angles. From postulate five, we can then say that the lines AB and CD will touch on the side of B and D. But this would contradict the premise that the lines AB and CD are parallel. Therefore AGH and GHD must be equal. The above explanation is the first instance of Postulate Five appearing in a proposition. But would this proof still hold if the definition of parallel lines were different? Do parallel lines have to be the only set of lines that do not intersect? Lobachevski uses an opening in Euclid's examination of parallel lines to assert his own theory.

"Originally, Euclid postulated that parallel lines are those lines which do not intersect at any given point. Lobachevski's theorem sixteen defines parallel lines as the last non-intersecting line or boundary line. He can assert this due to an [opening in Euclid's theory]. The uncertainty presumed in sixteen is

> 'whether the perpendicular AE is the only line which does not meet DC....' (Lobachevski, Theorem 16).

[In other word, a line lies between the Euclidean parallels which does not intersect DC in figure 1.] The main reason for the uncertainty or why this can be a valid contention is there is no definite proof confirming that parallel lines never meet or a line slightly less than 'parallel' will intersect. If it cannot be proved that it will definitely never touch, then it cannot be proved that a line a degree closer to the other line will not also be parallel. So knowing this uncertainty, Lobachevski expounds on his theory of parallels. He creates a line inclined towards DC named AG which does not intersected DC. He repeats this process until the boundary line is [assumed], AH. AH does not intersect DC and is the last one of its kind. Every line that is inclined closer to DC than AH will cut DC. The [above] includes the concept of the angle of parallelism. Essentially, it states that the angle has to be less than $1/2\pi$. If the angle was equal to $1/2\pi$, then the angle of parallelism would be 90 degrees, which is the definition given by Euclid. The angle of parallelism is not a specific number because the angle is different under distinct circumstances. This encompasses Lobachevski's definition of parallel lines." (*Difficulty Squared*, Price[16])

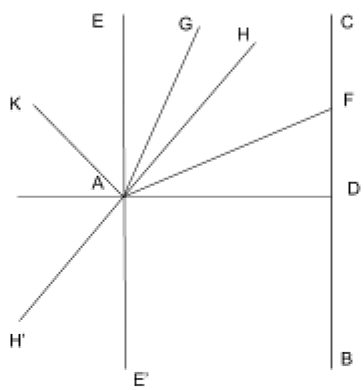


Figure 1

The two authors[17] disagree not just on the point of parallel lines but also about the sum of triangles. The disagreement on triangles also leads to many other problems including Lobachevski's construction of a "square" or square-like figure. "In Lobachevski, the reader [utilizes the] assumption that the sum of the angles in all triangles are less than π. In Euclidean

[16] Excerpt taken from a previous paper written by myself.
[17] Just for clarification, the two authors being Euclid and Lobachevski

geometry, triangles are equal to π. [π is equal to 180 degrees within this demonstration.] The problem caused is the sum of the angles in a Lobachevski square is less than a Euclidean square.

Figure 11α and 11β show the difference between the two sums. When the two halves or triangles that compose the Euclidean square are combined, it equals 2π. When the same is done to Lobachevski's rectangle or square, it equals 2π-α-β. (It does not matter whether it is a rectangle or square in this case because the outcome would be the same regardless.)

Figure 11α

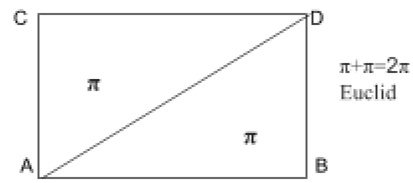
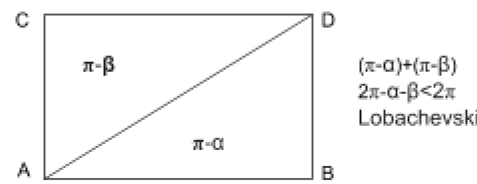


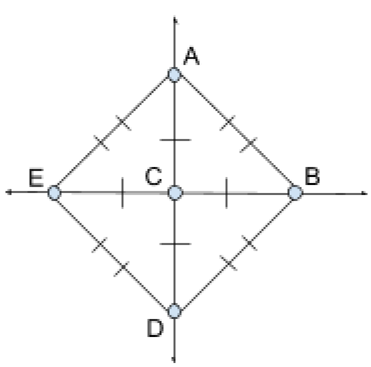
Figure 11β



But can this be resolved in the case of the non-parallel Lobachevski square? Again, maybe. If we take into account the reason why his triangles less than π. The main reason presents itself in theorem 22:

> '...in all rectilineal triangles the sum of the three angles is either π and at the same time also the parallel angles $\boldsymbol{\Pi}$(p)=1/2π for every line p, or for all triangles this sum is < π and at the same time also $\boldsymbol{\Pi}$(p)<1/2π' (Lobachevski, Theorem 22).

The first assumption is Euclid, while the second is Lobachevski. The justification for the triangles equaling less than π is connected with his definition of parallel lines" (*Difficulty Squared*, Price). A square-like figure can be achieved through a diamond-like figure, but the angles will still not be equal to 2π. In the figure is depicted to the left, AD and EB are equal straight lines which bisect each other perpendicularly. The lines AE, AB, BD, and DE are joined. All the angles at C are equal since they are all



$1/2\pi$ or right angles and AC=CD=CB=EC by construction. The triangles formed will then be equal to one another since they share a side-angle-side relationship. Thus we know AE=AB=BD=DE. The angles which subtend the equal sides are equal therefore the angle at A is equal to the angle at B, D, and E. Thus the above is a rectilinear figure with equal sides and equal angles. But since Lobachevski's triangles are $< \pi$, the figure adds up to $<2\pi$.

The above interpretations of parallel lines and triangles raises doubt as to the certainty of Truth in geometry. Euclid's nor Lobachevski's theory is neither technically more or less true, but both can be correct because they logically follow from their respective axioms. All that follows theorem 16 in Lobachevski comes from assumption that parallel lines are the last non-cutting line not the only non-cutting line. Everything which proceeds Euclid's explanation of parallel lines follows the assumption parallel lines are specific lines which are the only lines that do not intersect and have interior angles on the same side equal two right angles. On account of triangles, again, neither of the authors are incorrect. The premise for Euclid is the sum of the angles in a triangle equal two right angles and the proof thereafter logically follows this postulate. In similar fashion, Lobachevski's premise is the sum of the angles in a triangle equals less than two right angles and everything logically progresses from this axiom. But does the discrepancy between these two theories matter? Yes.

The above discrepancy exhibits the problem with mathematical truth being logical consistency. As seen in the etymology, truth should be stable, constant, and solid, but in the case of logical progression that cannot be said of the type of truth discovered. The truth found is particular not universal. In other words, it applies within the internal system of the logical progression itself. If the truth cannot be applied universally then is it really true? The theory of parallel lines founded by Euclid was able to be supplemented by the theory by Lobachevski and still works within his system. Parallel lines were a common notion within Euclid's geometry and it was something taken as truth or proven to be true through his proposition. But then Lobachevski came along and called his postulate into question. This just shows something taken as evident or necessary can be thrown into doubt. If this can be doubted what is to say other more complex theories cannot also be questioned? Also Socrates and Einstein said above, there is no real Truth for the geometry logical progression but most people would say it is something fundamental maybe even evident. Could we be wrong? Could Lobachevski be right about parallel lines? Could something as simple as this be called into question? So how would we fare when it comes to something as difficult as physics or the movement of the heavens? Now that we have seen the first definition of mathematical truth does not achieve or guarantee Truth, We can now look to the second definition for reassurances.

Ptolemy and His Theory of the Heavens

For the sake of clarification, Ptolemy's theory is classified under the second definition of mathematical truth: a theory being internally logical but also having correspondence to reality. Ptolemy is technically referred to as an astronomer, but he can also be considered a physicist. What is physics but the study of nature and laws governing the world? Ptolemy's theory sought to explain the movement of the "heavens" by not only utilizing logical progression but also empirical knowledge. His theory correlated to reality and accurately predicted the movement of the planets. All of which denotes Ptolemy can be characterized as a physicist or, in the very least, his theory falls under the second definition of mathematical truth.

So why investigate the second realm of mathematical truth: empirical mathematics? And what differentiates it from the first realm? As previously exhibited, the first realm of mathematical truth does not guarantee our common notions cannot be false. Einstein and Plato both assert internal consistency holds no real Truth since it does not correlate with reality. Therefore, we should examine a theory which utilizes empirical knowledge and gives the impression of certainty. If this type of theory can seem perfect in the time of which it originated but collapses when challenged by new information, then the question "could we be wrong?" is pertinent. If we were incorrect about erroneous theories in the past, we can be mistaken about common notions in the present or future.

Being justified in our research, we come to the work of Ptolemy. His theory is the finest test subject since, in his time, his argument was considered faultless, but, when challenged by Copernicus' theory, was replaced by said theory. Ptolemy's theory is based off false first principles that were agreed to be common knowledge and the opposite view was seen as preposterous by the general public[18].

Why are first principles important? First principles are like the foundation of a building. If the foundation is weak or unstable, then the whole building will collapse upon itself. Therefore, first principles have to be sound for the whole theory to be considered True. When people review Ptolemy's theory and observe the false first principles employed in the argument,

[18] There were some scientist/mathematicians who disagreed with his model but their theories were not generally accepted during this time. In his book, Ptolemy acknowledged these theories but believed them to be ridiculous. These passages will be hinted on later in this text.

they should take into account that first principles are not used in spite of their incorrectness but because they are supposed to be the common ideas of the time and seemed not to need an excessive explanation. But accepting notions as truths can be a risk leading to false conclusions and/or uncertainties. As we will see next, Ptolemy surmised the earth was in the center of the universe, which was a common belief at the time, but since it is now conceived not to be the center, the foundation for his theory is incorrect.

So what were his first principles? In the first eight sections of Book One of *Ptolemy's Almagest*, Ptolemy sets forth his first principles as theorems. In each section, his phrasing hardly disguises the fact he believes the first principles are obvious or cannot be thought otherwise. For instance, in premise three "That heavens move like a sphere" the audience is given a synopsis of why the belief of the heavens moving about the earth in a circular motion is accurate.

> "They saw that the sun, moon and other stars were carried from east to west along circles which were always parallel to each other...What chiefly led them to the concept of a sphere was the revolution of the ever-visible stars, which was observed to be circular, and always taking place about one centre...if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upward, must vary...the following considerations also lead us to the concept of the sphericity of the heavens. No other hypothesis but this can explain how sundial constructions produce correct results…" (H10-13).

Ptolemy's argument is centered around empirical evidence from the time before him and the contemporary time. He also goes through a thought experiment assuming the heavens move in other motions other than circular which leads to a contradiction with reality. Essentially saying, it is absurd[19] if the idea of the heavens moving like a sphere was incorrect. The language above is indicative of a common notion being defended against other theories which challenge the idea.

[19] Absurd, inasmuch as, it cannot be thought otherwise than how the current time assesses the situation.

His second premise was headed: “That the earth is in the middle of the heavens”. In similar fashion to the last, the following premise also defends the common notion at the time the earth is in the center of the universe.

> “...if one next considers the position of the earth, one will find that the phenomena associated with it could take place only if we assume that it is in the middle of the heavens, like the centre of a sphere. For if this were not the case, the earth would have to be either [a] not one the axis [of the universe] but equidistant from both poles, or [b] on the axis but removed towards one of the poles, or [c] neither on the axis not equidistant from both poles...if the earth did not lie in the middle [of the universe], the whole order of things which we observe in the increase and decrease of the length of daylight would be fundamentally upset.” (H17-19).

Again, Ptolemy’s argument hinges on empirical evidence and common notions. In a way, it is an understandable miscalculation to conclude the earth is in the center of the cosmos. During the time, the evidence utilized was through observation and no other generally accepted premise was surmised. Regardless, the above is how we could be wrong in our mathematical understanding. Common notions are trusted because they cannot reasoned otherwise with the information at the designated moment. What other concrete evidence could he have used to prove otherwise? Common notions are utilized because they are perceived to be obvious, proven ideas and anything to the contrary is preposterous. Ptolemy exercised the above style of argument because the idea of the earth being anywhere but at the center of the cosmos seemed absurd. In addition, he believed to have proved the former statement because if the earth was not at the center

> “the whole order of things which we observe in the increase and decrease of the length of daylight would be fundamentally upset” (H19).

All we *observe* about the cosmos would not correlate to the premise of the earth being off center. Common notions such as the former one appear evident but it does not guarantee Truth.

Appearances do not always correlate to physical existence in actuality. The earth, in fact, does not inhabit the center of the universe, but this theory was generally accepted for over 1300 years until adequately challenged by Copernicus.

There is one more premise I would like to discus before I move on: Ptolemy's first principle "That the earth does not have any motion from place to place, either". If we assume that the earth is at the center it logically follows it would have no motion. In his argument Ptolemy says:

> "One can show by the same arguments as the preceding that the earth cannot have any motion in the aforementioned directions, or indeed ever move at all from its position at the center...If the earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size: living things and individual heavy objects would be left behind, riding on the air, and the earth itself would very soon have fallen completely out of the heavens. But such things are utterly ridiculous merely to think of" (H21,24).

In this case, the common notion of the earth does not move is the consequence of the notion of the earth being the center of the cosmos. If we assume the later, we have to agree to the former. Ptolemy hypothesizes: it would be preposterous if the earth had motion since objects that were not tied down to the earth would be flung through the air. The argument is exceedingly near to a reduction ad absurdum: imagine if the earth was in motion. If it was then nothing could stay grounded unless attached to the ground. Since the above is not the case, the earth is stationary. In addition, if the earth was not stationary, the calculations and observations performed under the assumption the earth is at the center of the universe would be upset. Hypothesizing to the contrary would be found to be nonsensical empirically.

In the same section as the formerly mentioned except, Ptolemy entertains the hypothesis contrary to his. The hypothesis in which some believe the earth is not the center of the cosmos but instead the sun is and the earth is in motion. Ptolemy comments:

> "...they suppose the heavens to remain motionless, and the earth to revolve from west to east about the same axis [as the heavens], making approximately one revolution each day...from what would occur here on earth and in the air, one can see that such a motion is quite ridiculous...they would have to admit that the revolving motion of the earth must be the most violent of all motions associated with it, seeing that it makes one revolution in such a short time; the result would be that all objects not actually standing on the earth would appear to have the same motion, opposite to that of the earth…" (H24-25).

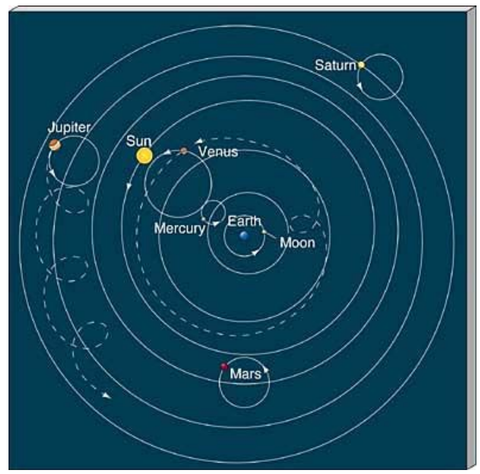
He again mentions if it were so that the earth was in motion, untethered things would be sure to be flying through the air. He never fully entertains this line of reasoning because for him the assertion is preposterous. Therefore, the earth cannot have motion and is in the center of the heavens. This also shows the unwillingness of the majority of people during the time to contemplate any first principles except those considered common notions. This prejudice against new and different ideas is not a new phenomenon. Common notions appear as infallible ideas to the people of the time while anything different seems nonsensical. Common notions are, indeed, not unerring. Mistakes are made and our perceptions of the world are not always accurate.

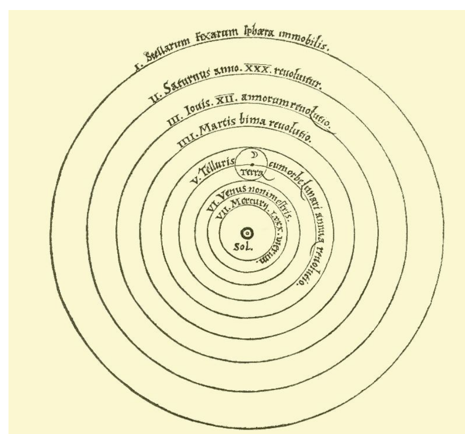
So to sum up Ptolemy's first principles used in the *Almagest*, the earth is motionless at practically the center of the cosmos while the heavens move around it in circular motion. In his own words:

> "...the heaven is spherical in shape, and moves as a sphere; the earth too is sensibly spherical in shape, when taken as a whole; in position it lies in the middle of the heavens very much like its center; in size and distance it has the ratio of a point to the sphere of the fixed stars; and it has no motion from place to place" (Ptolemy, H9-10).

He supposed anything to the contrary to the above common notions to be nonsensical. But, in a turn of events, we currently believe all his first principles to be ridiculous.

The shift in the vision of the cosmos is explained in more detail by the renowned physicist Stephen Hawking:





> "Ptolemy's model [depicted on the right] provided a reasonably accurate system for predicting the positions of heavenly bodies in the sky. But in order to predict these positions correctly, Ptolemy had to make an assumption that the moon followed a path that sometimes brought it twice as close to the earth as at other times. And that meant that the moon ought sometimes to appear twice as big as at other times! Ptolemy recognized this flaw, but nevertheless his model was generally, although not universally, accepted. It was adopted by the Christian church as the picture of the universe that was in accordance with Scripture, for it had the great advantage that it left lots of room outside the sphere of fixed stars for heaven and hell. A simpler model [depicted to the right], however, was proposed in 1514 by a Polish priest, Nicholas Copernicus. (At first, perhaps for fear of being branded a heretic by his church, Copernicus circulated his model anonymously.) His idea was that the sun was stationary at the center and that the earth and the planets moved in circular orbits around the sun [the theory commonly known as the heliocentric model]. Nearly a century passed before this idea was taken seriously. Then two astronomers – the German, Johannes Kepler, and the Italian, Galileo Galilei – started publicly to support the Copernican theory, despite the fact that the orbits it predicted did not quite match the ones observed. The death blow to the Aristotelian/Ptolemaic theory came in 1609. In that year, Galileo started observing the night sky with a telescope, which had just been invented. When he looked at the planet Jupiter, Galileo found that it was accompanied by several small satellites or moons that orbited around it. This implied that everything did not have to

> orbit directly around the earth, as Aristotle and Ptolemy had thought." (*A Brief History of Time*, Hawking 3-4).

The current common notion is thus rooted in the heliocentric model. After Copernicus, many other mathematicians/physicist have agreed with the model of heliocentricity making it the new common notion. But who is to say the common notion will not also be debunked in the same fashion as Ptolemy's theory? Our calculations proving the current theory may seem perfect but so did Ptolemy's at the time. But what if we are wrong? It is improbable we are correct on every account of our mathematical understanding. How will we ever know when our fundamental beliefs achieve real Truth? If there is no way to say definitely we are 100% correct about all our mathematical theories, who is to say that we are not the Ptolemy's of our time?

Ptolemy was correct under empirical mathematics but that did not guarantee that one day his theory could not be disprove. We have since proven the invalidity of the geocentric theory causing it to seem nonsensical. All the calculations can be correct, it can correlate to reality, but it can still be wrong. In this case the first principles were taken as truth because they could not be proven otherwise which caused the whole theory to be false. At the time, the calculations and observations appeared nearly flawless but with more information the theory was easily invalidated. So what is our saving grace if neither the two definitions of mathematical truth guarantee real Truth. How could we know we are definitely right? This essay has no answers on this front. Potentially, anything we believe could technically be false. But you may be asking yourself: Sure, but I still cannot understand how we could be so obviously wrong and not know it?

Concluding Thoughts

How could we be wrong without knowing it? Similarly to Ptolemy's theory, our commonly held theories are based off information hypothesized or assumed because they could be thought otherwise. In Ptolemy's case, the earth is in the center of the universe was this common idea which "could not be thought otherwise". As Ptolemy pronounced: it would be absurd[20] to think the earth was anywhere but at the center of the heavens and in any type of motion. On the first day of Freshmen Laboratory, the question is usually asked: what accounts for the motion of the sun across our sky everyday? The common response is: we revolve around the sun causing it to appear as if it were rising and setting everyday. But when asked to explain the above statement students reply with various reasons: maybe NASA or common knowledge. Then when asked to prove the earth is moving and not the sun, the challenge arises. How do we prove it only using empirical knowledge[21] and data? For a freshmen in their first Laboratory class, it can be practically impossible to confirm our commonly accepted theory[22]. Testing our common knowledge is part of the integral way. Why do we believe ____? How can we prove ____? Where does the original theory come from? What evidence do we have? What do the mathematicians say? How did they theorize about it with the information they were given? So, how could we be wrong without knowing it? We could decide not to test[23] everything as an individual. Expecting those who come before us to be correct in their proof or hypotheses concerning our common notions. When they say they have proven a theory we take their word.

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[20] Absurd: the word which denotes something which cannot be otherwise but the general opinion.
[21] Our observations.
[22] The theory the earth revolves around the sun.
[23] Test meaning: we do not research our generally agreed upon ideas to figure out where they came from and how they were "proven" or justified.

But testing our knowledge is how innovation and improvements happen. Why did Copernicus come up with his theory of the heavens where the sun is at the center of the cosmos? He challenged the common notions at the time (Ptolemy's theory) and came up with a new, more true[24] version of the cosmos.

So again, how could we be wrong and not realize it? We could be complacent and not challenge theories that appear to be sheer perfection. Ptolemy was taken as truth for more than 1300 years before the theory was adequately challenged. We could condemn those people with new theories of the world and not give them the time of day. Why think through new challenges when the old idea works well enough? Why be open-minded when the old way is better?[25]

So for the last time: how could we be wrong and not know? We could be under the assumption that everything in the past which was discovered was correct and forget how we came to know the information. The Great Books Program has a specialized way of making students understand where our common knowledge originates. Where common sayings or references emerge. Integral causes students to challenge and argue through the theories of the past mathematicians, physicists, astronomers, philosophers, psychologists, playwrights, poets, and great thinkers. Once we forget where our knowledge comes from, we forget why they are commonplace beliefs.

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[24] What do I mean by "more true"? I mean, simpler. Because in mathematics, simplicity equals more accurate. Why is Newton considered more accurate than those before him? His theory explains the world in a simpler, more accurate fashion than any other theory.

[25] I say this with the Christian church in mind since they have a history of supporting people because of their own agenda. As Stephen Hawking said in the lengthy quote above: "It was adopted by the Christian church as the picture of the universe that was in accordance with Scripture, for it had the great advantage that it left lots of room outside the sphere of fixed stars for heaven and hell." (Hawking).

So what is the solution to being wrong? I am not here to provide assurances but we have been adapting since the beginning. It is okay to have unresolved issues about our world. It makes us strive for something. Drives us to discover something bigger than just ourselves. Progress as humans. Advance as a society. Theories may be disproven but that does not mean our physical world is going to change. New theories might come forward which will fit more perfectly with empirical evidence making our world more understood, but life will always go on and theories will give us a greater understanding of our universe.

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