From Descartes to Kant: Mathematics, Certainty and the World

By

Rex B. Rafanelli

A Senior Essay submitted in partial fulfillment of the requirements for the degree Bachelor of Arts in the Integral Curriculum of Liberal Arts

_____

Steve Cortright, Advisor

Saint Mary's College of California

May 8, 2015

General Introduction

The aim of this essay is to examine and compare the philosophical projects of René Descartes and Immanuel Kant. To an extent both philosophical projects can be seen as an attempt at answering the question of why mathematics possesses certainty. Furthermore, their respective answers to this question end up influencing their broader philosophical projects to a surprising degree.

As we will see, this is because the question of how mathematics possesses certainty invariably leads to the question of how mathematics relates to the world; and this question, in turn, determines the answer to an even more fundamental question of what kind of relationship our minds have to the world. For Descartes and Kant all of these questions are inherently interconnected, and thus they cannot answer one without, in some way, answering the others. In the effort to answer the above questions a historical development of ideas arises. When the answers provided by Descartes fall short, Kant's project advances with a more comprehensive and systematic solution. Thus we now commence with Descartes, the first of our philosophers, in hopes of discovering where it all began.

An Introduction to the Cartesian Project

Descartes was a peculiar kind of genius, though not uncommon for his time. He is counted among the great polymaths of Western thought, earning this title because his interests were as numerous as they were diverse, and that's not to say he dabbled in a number of subjects, but that he authored extensive works in these subjects, some of which had a significant influence. The most successful of his contributions were made in the field of mathematics. For it is here,

perhaps more than anywhere else, that one witnesses the height of Cartesian ingenuity. In particular, I am thinking of Descartes' solution to the advanced locus problems of geometry—I say *solution* instead of *solutions* because rather than solving a number of individual problems Descartes devised an entirely new kind of hybrid mathematics, he called analytic-geometry, that served to bridge the gap between geometry and algebra and allowed him to treat the whole class of locus problems generally. While Descartes' mathematical works might have been his most widely considered successes, they were not his most influential. Though, as we will see, they certainly did influence his philosophical project, which holds the title of most influential work.

But truth be told, one does not have to be correct to be influential. The philosophy of Descartes, as the foundation of the Cartesian Project, eventually gave rise to many ideas that were neither correct, nor influential. To observe the dramatic disparity all one must do is to read Descartes' theory describing the motion of the blood and the function of the heart. One wonders how such a brilliant thinker could stray so far from reason, common sense and truth—*he had read Harvey for Pete's sake!*(Descartes 76)

Of course this isn't the whole story, as was alluded to above. Just like many other polymaths Descartes was remarkably ambitious, with the intention of developing a complete and comprehensive world-system. It's important to understand this point, for to see his theory on the heart and his analytic-geometry as unrelated is to miss the true character of the Cartesian Project, not to mention Descartes himself, who on this view appears to owe his ideas to some kind of divine madness rather than reason of his own. But Descartes would roll over in his grave if he heard such a thing! There's a reason Descartes philosophy is called a rationalism, for he wanted nothing more than to develop human reason to its highest perfection, to know all that can be

known. Little did Descartes think that he would stray from the true path. Thus he unknowingly described himself when he said, “The greatest minds, as they are capable of the highest excellences, are open likewise to the greatest aberrations.”(Descartes 39) In the following section we will see exactly why he was so confident in his project, and why he thought he had developed a method that could not fail.

Origin of the New Method

Descartes’ project is initiated by the transformative discovery that, after all he had learned in his many years of formal education, it was quite possible he had been left empty handed. What’s more, he thinks that if his hands are empty, then ours are too. The problem of concern is most apparent in philosophy. What Descartes has observed is that the development of philosophy seems to have corresponded to an increase in uncertainty. To him, the so called progress of philosophy is nothing more than a proliferation of uncertain ideas. The very fact that so many diverse, and often contrary, ideas are allowed to stand supports this view.

> When I considered the number of conflicting opinions touching a single matter that may be upheld by learned men, while there can be only one true, I reckoned as well-nigh false all that was only probable.(Descartes 44)

For every philosopher has an antagonist who doubts what he asserts; so long as there can be disagreement there can be doubt. Thus it appears the very fabric of philosophy is woven from doubt—a great blanket of uncertainty.

Descartes is primarily concerned with philosophy because it possesses priority in two important senses. The first, according to Descartes, is that philosophy is the foundation of all the other sciences. In this way the sciences depend upon philosophy and can be no better off than the

foundation that philosophy provides. The second is closely related to this point; it is philosophy's priority in time. Emerging long before the other sciences, philosophy has a head start. Accordingly, one expects, like Descartes, that philosophy would have made significant progress in this time. Descartes concludes that all of philosophy is plagued by an unacceptable degree of uncertainty, and furthermore, because the sciences depend on their philosophic foundations, they too, carry with them this fatal flaw.

> Of philosophy I will say nothing, except that when I saw that it had been cultivated for many ages by the most distinguished men, and that yet there is not a single matter within its sphere which is not still in dispute, and nothing, therefore, which is above doubt, I did not presume to anticipate that my success would be greater than that of others […] As to the other sciences, inasmuch as these borrow their principles from philosophy, I judged that no solid superstructures could be reared on foundations so infirm.(Descartes 44)

Skepticism has its place in Descartes' philosophy, but it only plays a minor part in his overall project. It must be emphasized that it is the mere *possibility* of doubt that is the worry for Descartes. Thus one is wrong to call Descartes a skeptic. For Descartes only intends to entertain a skepticism as a means to an end; it is at most a methodological skepticism, and it's aim is a kind of purification of the mind. With this method Descartes hopes to seek a new foundation for philosophy. And indeed he does, with his famous one-liner, "*Cogito ergo sum*: I think therefore I am."(Descartes 63)[1] But as famous as this expression is, it does not express the true character of Descartes' project, at least not without adding some points of clarification. The reality is that Descartes entertains a provisional skepticism. This means that even when dabbling in doubt Descartes still has a deep-seated faith that reason will see him through.

> As for the opinions which up to that time I had embraced, I thought that I could not do better than resolve at once to sweep them wholly away, that I might

[1] From here forward Descartes' *cogito ergo sum* will simply be referred to as the *Cogito*.

> afterwards be in a position to admit either others more correct, or even perhaps the same when they had undergone the scrutiny of reason.(Descartes 48)

The skepticism is more of a form of mental purification than anything else. Descartes' trust in reason certainly clashes with the view that he constantly does battle with a relentless skepticism. Though it's understandable how one could be misled after seeing the dramatic defeat of such extreme skepticism at the hands of the *Cogito*.[2]

It is essential to point out the disconnect between the *Cogito* and the rest of Descartes project. Though in the course of his works many more discoveries follow the *Cogito*, none follow from it in the logical sense. For the *Cogito* provides nothing except the knowledge that its subject must exist. But what good is that? No one wants to exist all alone. What Descartes really wants is to affirm the world; and when eventually he does it's only through his trust in reason.

Mathematical Inspiration: Cartesian Certainty

At the heart of Descartes' project is the search for certainty, but the particular nature of this certainty has yet to be identified. Looking back and recalling Descartes' earlier success in the field of mathematics, we may begin to get an idea as to the kind of certainty he has in mind. Descartes is by no means hiding this inspiration from us. At the very outset he expresses his overwhelming satisfaction with mathematics.

> I was especially delighted with the mathematics, on account of the certitude and evidence of their reasonings […] I was astonished that foundations, so strong and solid, should have had no loftier superstructure reared on them.(Descartes 51)

[2] Extreme Skeptic: "Nothing exists, not even you!" Descartes: "Wrong! I at least know that I exist insofar as I am a thinking thing."

Of course it's mathematical certainty that Descartes wants; he is a mathematician after all. Descartes' goal, then, is to extend the coercive necessity of mathematical inference to the subjects of philosophy and science. He proceeds with complete confidence, never considering that philosophy, or science for that matter, may be incapable of the kind of absolute certainty that abounds in mathematics. Descartes' first-hand experience with the nature of mathematics has made him exceedingly hopeful for the future of human reason.

> The long chains of simple and easy reasonings by means of which geometers are accustomed to reach the conclusions of their most difficult demonstrations, had led me to imagine that all things, to the knowledge of which man is competent, are mutually connected in the same way, and that there is nothing so far removed from us as to be beyond our reach, or so hidden that we cannot discover it. (Descartes 52)

The nature of the Cartesian Project is far more mathematical than skeptical. While at the outset a provisional doubt allowed for a kind of purification of the mind, it was only a means to *this* end. To be sure, Descartes was well acquainted with the certainty of mathematics long before he emptied his mind of all of its beliefs. For Descartes, mathematics stands as the shining example of what properly conducted human reason can accomplish. This is why only the first of his rules for method pertains to doubt, while the other three have to do with to certainty. Thus it is from mathematics that Descartes philosophy truly emerges.(Descartes 51)

But exactly how does Descartes hope to extend this particular certainty beyond mathematics? The approach is so simple Descartes could have conceived of it as a child: "I like what happens in math, can we do the other ones like that too?" To which a reasonable response would be, "Yes, that's a fine idea Descartes! But what's needed first is to figure out exactly what happens in mathematics that makes it special, then, if possible, we can make the others special

like it too." In order to figure out how to apply the certainty of mathematics to philosophy and science, he first must to figure out what sets mathematics apart. How do we account for the certainty of mathematics? If Descartes hopes to extend mathematical certainty beyond the realm of pure mathematics he must provide an answer. Here's how it works: Descartes reasons that the certainty of mathematics must come from the particular nature of the objects that it studies, namely, numbers, lines, figures, and the like.

> And I had little difficulty in determining the objects with which it was necessary to commence, for I was already persuaded that it must be with the simplest and easiest to know, and, considering that of all those who have hitherto sought truth in the sciences, the mathematicians alone have been able to find any demonstrations, that is, any certain and evident reasons, I did not doubt but that such must have been the rule of their investigations.(Descartes 52)

Thus the certainty of mathematics is not particular to mathematics, but rather, it comes from a more general principle that is followed by mathematics. The *principle of simplicity*, as we will call it, may be exemplified by mathematics, but is by no means limited to it. The inspiration for this move is taken right out of Descartes' analytic-geometry, where he says that the simple objects of mathematics must be applied to every other class of objects whenever it be possible.(Descartes 52) Furthermore, he believes the method is general enough to accomplish this. "Not having restricted this method to any particular matter, to [attempt to] apply it to the difficulties of the other sciences."(Descartes 54) But now we are left with the further question of how we are to determine exactly what objects qualify for mathematical treatment, and this brings us to the next section.

The Object of Geometers

The first discussion of the *geometer's objects*, though brief, occurs in *A Discourse on Method.* It's glossed over as Descartes is merely using it as an example of something that is clearly and distinctly perceived. Nevertheless, Descartes' remarks are rather telling; they give us a glimpse of what's to come. Consider the relationship between space and geometry here:

> The object of the geometers, which I conceived to be a continuous body, or a space indefinitely extended in length, breadth, and height or depth.(Descartes 65)

Here Descartes proposes that space is essentially geometrical. We will see soon how far he intends to take this line of thinking.

Turning to Descartes' subsequent work, *Meditations on First Philosophy*, we find him considering what is commonly thought to be distinctly known, that is, corporeal bodies or bodies of sense. He proposes a curious thought experiment where we are asked to contemplate a single particular piece of wax. The first thing we notice is that the piece of wax has many distinct qualities: colour, figure, size, smell, texture, temperature and even a particular sound when it is tapped upon. It's likely that we agree with Descartes when he says, "In fine, all that contributes to make a body as distinctly known as possible, is found in the one before us."(Descartes 124) But what happens when the piece of wax is placed near the fire?

> What remained of the taste exhales, the smell evaporates, the colour changes, its figure is destroyed, its size increases, it becomes liquid, it grows hot, it can hardly be handled, and, although struck upon, it emits no sound. Does the same wax still remain after this change? It must be admitted that it does remain; no one doubts it, or judges otherwise. What, then, was it I knew which so much distinctness in the piece of wax? Assuredly, it could be nothing of all that I observed by means of the senses, since all the things that fell under taste, sight, smell, touch, and hearing are changed, and yet the same wax remains.(Descartes 124)

Therefore Descartes concludes that the wax was none of these things perceived previously by the senses. With all these qualities gone we are left with the question: what could be left of the wax? There is just one thing that remains, and that thing is *extension*. But, Descartes continues, if the wax is truly just extension, then the wax has the ability to make an infinite number of alterations in size and shape. As it turns out, this far surpasses our mind's capacity for representation that is called imagination.

> Unable to compass this infinity by imagination, and consequently this conception which I have of the wax is not the product of the faculty of imagination … I must, therefore, admit that I cannot even comprehend by imagination what the piece of wax is, and that it is the mind alone which perceives it.(Descartes 124)

It is a peculiar faculty of mind, then, that can represent the wax in its infinitely many diverse appearances and still have it be understood as pure extension. Descartes states that it is merely an "intuition" of the mind.(Descartes 125) Furthermore, with this one aspect of the wax being now the only thing that is clearly and distinctly perceived in it, we have assurance that, in this respect, it truly exists. That is, by way of the *principle of simplicity,* we know that out of all the things that appear to be outside us, at the very least, their geometrical aspects really exist. It follows from this that these corporeal objects are actually and essentially geometric. Descartes says this himself, "what is remarked of this piece of wax is applicable to all the other things that are external to me."(Descartes 125)

But what is to become of those other qualities first perceived in the wax? Those qualities corresponding to smell, touch, sight, hearing, and so on. Are they merely illusions? It's hard to say, for all that is known for sure is that they have faded into uncertainty. Descartes says,

> I cannot determine even whether they are true or false; in other words, whether or not the ideas I have of these qualities are in truth the ideas of real objects.(Descartes 136)

The ideas represented by the senses fall into doubt. Unlike the geometric extension of these bodies which is clearly and distinctly perceived, and is therefore real, these representations from sense may lack objective reality. Thus, if they do not proceed from real objects, then where can the come from but within us?

An Introduction to the Kantian Project

Immanuel Kant's project is very similar to that of Descartes. At a fundamental level both are dealing with the domain of reason: Descartes hopes to extend its domain indefinitely while Kant hopes to properly determine it. Both projects attempt to refound philosophy and cast away everything that comes before them. While they are similar in this way, they do have slightly differing aims, this is largely due to their respective places in history. Descartes sought to refound philosophy with the intention of providing the sciences with a new foundation, upon which they could flourish. While his project as a whole might have failed, his mathematical successes as well as his geometrical treatment of space and corporeal objects were ideas central to the scientific revolution. To that extent his philosophy was both successful and influential. When Kant arrives on the scene the stunning success of the scientific revolution is undeniable, his task is clear: reverse engineer philosophic foundations for the already flourishing sciences.

Kant's work is largely a response to the skeptical philosophy of David Hume.(Kant B20) Though Kant's project is initiated by skepticism, it is much different from the skepticism of Descartes: it requires no *Cogito* and is in no way methodological. Thus one might be inclined to call this skepticism genuine. The skepticism faced by Kant, that is, the skepticism put forward by David Hume, is one that Kant believes undermines reason by calling into question the necessary

connection between cause and effect.(Kant B20) While the consequences of this skepticism are probably not altogether obvious, for the purposes of our investigation we can put the particulars aside and simply say that Kant believes that the laws of Newton's physics cannot stand if Hume is correct. Thus he attempts to address the problem of Hume more generally.[3] Under Kant's analysis, Hume's problem of the necessary connection is seen to have even farther reaching consequences to include not only science but also our beloved mathematics—as we will see, all *a priori* synthetic judgements are on the line.

For Kant there is no question about it, science and mathematics are doing something right. Taking as given the success of science and mathematics, the question is not *if* they are possible, but rather *how* they are possible as we see them before us. For Kant, and most of us, there is no doubt that they are possible, for they are already actual. We saw similar inspiration in Descartes; when with the certainty of mathematics in mind he sought to refound philosophy. Now, when Kant attempts this same move, he does so with a view to mathematics even more generally.(Kant B20) "The real problem of pure reason is now contained in the question: How are synthetic judgements *a priori* possible?"(Kant B19) Of course, we are not yet familiar with Kant's technical language and so the above quotation is all but meaningless. There's no need to worry, however, for understanding the meaning of this central question is our primary purpose in the pages that lie ahead.

Fortunately it is a manageable undertaking. This question does not require we understand the totality of Kant's *Critique*, we only need to understand its first principles, so to speak. To

[3] Hume does not believe *a priori* synthetic judgements are possible. According to Hume, causes are conceptually distinct from their effects and therefore separable, they are not analytic and therefore have no ground for necessity.

make this a less daunting task, most of what Kant has to say about this problem can be found in the twenty or so pages of *The Transcendental Aesthetic*. For Kant, answering the main question of reason begins with a new understanding of mathematics. However, this requires first that we become familiar with two conceptual distinctions fundamental to Kant's philosophy: the first are called *a priori* and *a posteriori* cognitions; and the second are called *analytic* and *synthetic* judgements.[4] It cannot be emphasized enough how fundamental these distinctions are to understanding Kant's philosophy; as important as they are, it is to be expected that we meet with them at the very outset, in the introduction to the *Critique*.

Fundamental Conceptual Distinctions

In the very first line of the introduction Kant affirms that all cognitions begin with experience. He then adds, "although all our cognition commences with experience, yet it does not on that account all arise from experience." (Kant B2) It is clear, then, that all of our cognitions are initiated by experience, but even so, the *possibility* still remains that upon being prompted by experience our cognitive faculties are not merely passive, but rather actively add something to our cognition. In this way our cognition would be composed of two parts, that is, what came from outside us and what came from within us to meet with it. If this is true, then part of our cognition arises from experience and the other part arises from us.

Those cognitions that arise from within us Kant calls *a priori* and those that arise from experience he calls *a posteriori*. Though at first Kant speaks tentatively—of only the mere

[4] The term "judgement" will be used interchangeably with the term "proposition" in this context. The slight difference is merely grammatical: when one makes a judgement what they are doing is predicating something of a subject, thus every judgment is a proposition and every proposition a kind of judgement.

possibility of these *a priori* cognitions—it quickly becomes apparent that they do, in fact, exist. The title of the following section is "We are in possession of certain *a priori* cognitions, and even the common understanding is never without them." Kant elaborates on the distinction between *a priori* and *a posteriori* cognitions, noting that all cognitions of experience are lacking a certain quality, and that this quality is certainty.

To give an example, an ornithologist might say with great confidence "of course all swans are white!" But the very next day it is completely possible that this same ornithologist may return from the field (or lake in this case), head hung low, and say to us with overwhelming disappointment, "I saw a black swan today." What the ornithologist meant the day before, then, though it was unstated, is that "all swans that I have observed thus far have been white." That's the best one can do when the source of these judgements is experience. With new experiences always occurring, empirical judgements forever remain contingent upon future experiences, and thus never reach the level of strict universality. Unfortunately for the ornithologist there is always the possibility that something contrary may be observed. With this established we understand that where necessity and strict universality *are* present we cannot be dealing with experience, and therefore, we know that we are dealing with *a priori* cognitions.

> Thus if a judgement is thought in strict universality, i.e., in such a way that no exception at all is allowed […] then it is not derived from experience, but is rather valid absolutely *a priori* […] this [universality] points to a special source of cognition for it, namely a faculty of *a priori* cognition. (Kant B4)

The ornithologist, now wise to his earlier mistake, might say, "Perhaps not all swans are white, but I know for certain that all swans are birds! *I am an ornithologist after all.*" The latter half of

the first statement possesses strict universality—our friend has caught on. This brings us to our next conceptual distinction, where we will see the two kinds of *a priori* cognitions.

But first, to quickly summarize what has just been covered. All cognitions are initiated by experience, but that is not to say all arise from experience, or, in other words, have experience as their source. The possibility stands that, being prompted by experience, there are cognitions that arise from within us. We also understand that cognitions from experience always lack necessity, and thus strict universality, but we know that we do make judgements of strict universality, like "all swans are birds", and so we affirm that cognitions arise from within us.

The Two Kinds of *A Priori* Judgements

"In all judgements in which the relation of subject to the predicate is thought […] this relation is possible in two different ways."(Kant B10) These two different ways consist of the two kinds of *a priori* judgments. Commencing with the simplest of the two we have the kind of judgements which Kant calls *analytic*. We already have a clear example with the ornithologist's most recent remark, "All swans are birds." It's easy to see that this is a statement where the term "swan" has been defined as a "bird." "Analytic judgements (affirmative ones) are thus those in which the connection of the predicate is thought through identity."(Kant B10) The language and phrasing here is a bit unusual, but what is meant is actually very simple: to ask the identity of a thing is simply ask what a thing *is*, and that's precisely what the definition does in the above case. If the ornithologist understands what is signified by the term "swan" he understands that a swan is a bird. Considering the ornithologist's statement as a proposition we use slightly different terminology, though the result is the same: the predicate (bird) is contained within the subject

(swan). Thus, these kinds of judgements can also be called "judgements of clarification" because through them the subject is broken up "by means of analysis into its component concepts, which were already thought in it."(Kant B11) This is why you cannot think swan without also thinking bird.

Now we come to the other kind of *a priori* judgement, and it's slightly more complicated than the first. We saw a moment ago that analytic judgements were about clarification, that is, looking into what is covertly contained within a given concept. The potential uses of analytic judgements are exhausted in mere clarification of concepts, and so they are not useful for much. While the next kind of *a priori* judgement is a bit more complicated, it is also much more useful. Kant names these *synthetic* judgements, calling them "judgements of amplification" because they are what allow for our cognition to relate concepts that are not related through identity. In other words, what is special about synthetic judgements is that even though the predicate is not contained in the subject, as it would be in an analytic judgement, it is still nevertheless connected in some other way. We saw an example of one with the ornithologist's first claim "all swans are white." In this case these two concepts, swan and white, the subject is not connected to the predicate by anything other than the ornithologist's recurring experiences. While this was a synthetic judgement, for all judgements from experience are synthetic, we are now looking to understand synthetic judgements that are *a priori*.(Kant B11) This poses a special problem: in being *a priori* these judgements cannot appeal to experience, and thus this means of help is entirely lacking.(Kant B12) It must be remembered that the question of how these concepts are connected is precisely what Kant calls "the real problem of reason."(Kant B19) In the next

section we will give an exposition of what is perhaps the most striking and common kind of *a priori* synthetic judgements.

Mathematics as Synthetic

To restate, a judgement is *a priori* synthetic when it meets the criterion for *a priori* cognition, that is, when it is thought with necessity or strict universality,[5] and when it is *not* analytic because its predicate is not connected to the subject through identity, but is somehow, nevertheless, still connected. So if, in fact, *a priori* synthetic judgements are possible, the question of interest becomes where can we find them. Kant proudly declares that he is the first to notice that all mathematical judgements are *a priori* synthetic.(Kant B14) Now there's no doubt that mathematics as a whole is *a priori*, for as we have seen it is prized for its certainty,[6] but the further question of whether or not it is synthetic is more difficult to determine. It requires us to survey a handful of mathematical judgements and to put them to the test: is the predicate concept contained in the subject concept? If it's not, and the concepts are not related through identity, we have a found a synthetic judgement *a priori*.

Let's take Kant's first example, the proposition "7 + 5 = 12".(Kant B15) We ask the question, is the concept of twelve contained in the concept the sum of seven and five? Kant says it is not, for "if one considers it more closely, one finds that the concept of the sum of 7 and 5 contains nothing more than the unification of both numbers into a single one, through which it is not at all thought what this single number is which comprehends the two of them."(Kant B15) The point may not be so obvious with these smaller numbers because our minds perform these

[5] B4

[6]For our purposes certainty and necessity mean the same thing.

operations so quickly. However, Kant assures us that "one becomes all the more distinctly aware of that if one takes somewhat larger numbers, for it is then clear that, twist and turn the concepts as we will […] we could never find the sum by means of the mere analysis of concepts."(Kant B16) The case of geometry is far more obvious. Take a straight line for instance,

> That the straight line between two points is the shortest is a synthetic proposition. For my concept of *the straight* contains nothing of quantity, but only a quality. The concept of the shortest is therefore entirely additional to it, and cannot be extracted out of the concept of the straight line by any analysis.(Kant B16)

When Kant says "nothing of quantity, but only quality" he is referring to the line's straightness as its quality and the supposed shortness as its quantity. The concept of straightness does not contain within it the concept of shortness, thus it fails the analytic test.

Now we come to the central question of how *a priori* synthetic judgements are possible. At this point Kant refers us to what he calls the intuition, saying, "Help must here be gotten from intuition, by means of which alone the synthesis is possible."(Kant B16) If we can remember that with the synthetic judgement from experience we saw earlier (all swans are white), we noted that it was the experience itself that connects the two concepts, that is, it is the experience that makes the synthesis possible. In this same way, when the cognition makes synthetic judgements independently of experience it is the intuition that allows for the connection of the two concepts.

Space and Time as Transcendental

Long was it supposed that space and time were substances, or the relations of substances. If this is true, then both conceptions of time and space are empirical. But according to Kant, this cannot be the case, for not only are time and space *a priori,* but they are also *transcendental*.

This new term, transcendental, refers to that which is necessary for and thus makes possible synthetic judgements *a priori*, as well as experience in general. Kant says,

> I call all cognition transcendental that is occupied not so much with objects but rather with our mode of cognition of objects insofar as this is to be possible *a priori.*(Kant B25)

For experience, as it is, to be possible we must have *a priori* principles that determine and order the representations of objects that constitute this experience. The manifold of appearances, as Kant calls it, consists of the undetermined representations of objects from experience. These undetermined representations are nothing more than the confused, ever-changing and countless sensations that perpetually shower our senses during every moment of our existence. They need to be managed if there's any hope that they may rise to the level for cognition. What determines, orders and eventually renders intelligible these sensations are the *a priori* principles called the pure forms of intuition.[7]

It is understood that all our representations of experience are determined in space. When an object affects in us a representation, that object is always represented as being outside us.

> For in order for certain sensations to be related to something outside me, thus in order for me to represent them as outside and next to one another, not merely as different but as in different places, the representation of space must already be their ground.(Kant B38)

So long as space is supposed to be a substance, or the relation of substances, it is empirical; and thus it is represented as outside of us. To put it another way, *space is represented as in space*.

---

[7] The term intuition refers to that which relates cognitions to objects, and the term sensibility is the capacity for these objects to affect a representation.(Kant B34) Both are divided into two subcategories, intuition is divided into empirical intuitions and pure intuitions; sensibility is divided into the matter of appearances and the form of appearances (*but also called the form of sensibility*). At the very least what must be taken away from all this? **The pure form of intuition is the same thing as the pure form of sensibility,** that is all.

See the problem? To say such a thing is to beg the question. Space cannot be an empirical cognition because for it to proceed from outside of us is to already presuppose it. Therefore it is not only *a priori* but also transcendental because it is required for experience of things outside us altogether.

The above conclusion can also be derived in another way. By removing from an appearance (empirical representation) what is given by sensation, we are left with the pure form of intuition (*a process similar to that of Descartes' wax*). Let us now attempt this with space. In removing all sensible objects from our empirical representation we discover that, though space may be emptied of bodies, space itself cannot be made to disappear. Kant says, "One can never represent that there is no space, though one can very well think that there are no objects to be encountered in it."(Kant B38) Therefore the original representation of space is a pure form of intuition that makes possible the representations of experience.

But the main question of reason is not how experience is possible in general, but rather how synthetic *a priori* judgements are possible. The pure form of intuition is the solution. For the only way that geometry can determine the properties of space *a priori* and synthetically is if space and its properties are already given in the form of pure intuition. If we remember that for synthetic judgements of experience the subject is connected to the predicate through experience, we see that, similarly, it is possible to connect the subject to the predicate *a priori* only by means of the pure form of intuition.

Moving on to time, we again attempt to separate our representations into what is supplied by sensation and what is supplied from us. This case is more easily decided than the first. It's no trouble at all to remove appearances from time, but to try to remove time itself is impossible, for

all that is represented to us is represented in time. Therefore, time is a pure form of intuition. Furthermore, time is with us, even when space is not. For while space is the pure form of outer intuition, time is the pure form of inner intuition.

> Inner sense, by means of which the mind intuits itself, or its inner state […] is still a determinate form, under which the intuition of its inner state is alone possible, so that everything that belongs to the inner determination is represented in relations of time. Time can no more be intuited externally than space can be intuited as something within us.(Kant B37)

In a sense, then, time is prior to space, but only as cognition of ourselves is prior to cognition of external objects. Though not to be confused, both time and space are pure forms of intuition, and therefore transcendental. What is meant is this: for there to be an object as represented in space the concept of simultaneity is already supposed. Objects in space are always represented instantaneously, like a photograph. This is because as our form of inner intuition all representations are determined in time because the are represented in us. Different times are successive, while different spaces are simultaneous.(Kant B47) Thus it is not necessary for us to represent ourselves in space, but only in time.

Furthermore, it is only through time that alteration in general is possible, because contrary predicates may only be applied to a subject at different times, e.g., “The cat is small” and “The cat at is large” (for it has grown). Even the primitive concept of motion, predicates different places to a subject, e.g., “the bird is in the tree” and “the bird is not in the tree,” is only possible given succession in time.(Kant B48) Surely, then, time is an *a priori* condition for all experience in general.

In conclusion, time and space as taken together constitute all possible pure forms of sensible intuition.(Kant B56) How are we sure that there are no more pure forms of intuition?

> The transcendental aesthetic cannot contain more than these two elements, namely, space and time, is clear from the fact that all other concepts belonging to [intuition], even that of motion, which unites both elements, presuppose something empirical.(Kant B58)

In short, nothing is more fundamental, and thus every other concept from sense can be entirely reduced to just time and space.

To reiterate, we saw time and space cannot be substances or relations among substances as this would make them empirical, which would not only make experience itself impossible, but would also reduce geometry to a mere science of probabilities. The only way that any *a priori* synthetic propositions are possible is if space and time are the forms of intuition.

Perhaps we should elaborate on the one drawback to the transcendental, that is, that time and space only exist for us. To put it less technical language, they are merely the subjective constitutions of the human mind. The representations we receive are determined and organized by the forms of sensible intuition and they are all we have access to, and thus all we know. We know nothing about objects in themselves as they really are..

Final Thoughts

With a reasonable understanding of what both Kant and Descartes have to say about the certainty of mathematics, the relationship of mathematical objects to the world, and our capacity for knowledge of the world, at last we proceed to make observations and draw parallels.

On the subject of the certainty of mathematics Descartes appeals to what Kant would likely call “ordinary common sense.”(Kant 4:260) For upon reflection Descartes concludes that mathematics possesses certainty because the objects it studies are the most simple and knowable.

The further question of *why* the objects of mathematics are self-evident is not answered by Descartes, nor does he attempt to answer it. But that's not to say Descartes doesn't get close, in fact, he nearly stumbles upon the Kant's transcendental. Consider the thought experiment with the piece of wax. The wax presents to Descartes a manifold of appearances—and this remind us of Kant's manifold—he concludes that it is not the imagination that perceives the wax but the mind itself.[8] Doubtful of sense, Descartes removes all ideas that arise from sensation from the wax in order to know what the wax as it truly is. What he is left with is merely extension. Kant uses exactly the same process to separate sensations from the form of sensibility (otherwise called the pure form of intuition). Both come to the same conclusion, what's left is space (or extension). The obvious difference is that for Descartes what's left of the wax is really there in the world, which verges on idealism, whereas for Kant what remains is what is essential to our minds, and this is what he calls transcendental idealism. The former possess objective validity while the latter possesses subjective validity.

For Descartes the qualities of objects of sensation are divided into two distinct classes. Again the wax provides us a good example: upon examination Descartes finds that the sensible qualities of the wax are uncertain and thus cannot be said to correspond to objects. These uncertain qualities constitute everything that is not mathematical in the wax, namely, its color, taste, smell, temperature and the like. While the objective existence of these qualities is dubious, there is no doubt they exist in the mind of Descartes, and hence they possess subjective validity.[9]

---

[8] Descartes even calls it "intuition (*inspectio*) of the mind," meaning a sort of reflection on the mind.(Descartes 125) Though Kant would say that Descartes did not inspect thoroughly enough.

[9] Here is another, possibly clearer, example of these subjectively valid qualities: "And, though on approaching the fire I feel heat, and even pain on approaching it too closely, I have, however, from this no ground for holding that something resembling the heat I feel is in the fire, any more than that there is something similar to the pain; all that I have ground for believing is, that there is something in it, whatever it may be, which excites in me those sensations of heat or pain." (Descartes 169)

According to Descartes' line of thought, if the objects of mathematics were not objectively valid, that is, if they did not really exist outside of the human mind, then knowledge of everything but ourselves falls into uncertainty. This is not a problem for Kant, however, even though according to his account all objects of sensation are only known subjectively (exist for the subject). It's worth noting that this particular relationship of mind to object was introduced by Descartes. In his doubt regarding objects of sense the intimate relationship of mind to object is lost. No longer do we perceive things as they are in themselves, instead we have only representations; and we are left to wonder whether or not they arise from real objects at all.

When Descartes sees that everywhere he looks he cannot help but find geometry, he assumes that geometry is really out there. Space, therefore, becomes essentially geometrical. The latter is true for Kant as well, and this correspondence, present in both of their accounts, is what establishes geometry as *a priori.* When Kant looks around, however, and sees that everything outside of him is essentially geometrical, he concludes instead, not anything about the world, but rather something about himself. Descartes concludes this only for those objects of sense, as we saw in the wax. The difference between Kant and Descartes is analogous to the difference between Ptolemy and Copernicus. Thus, if it were possible, Kant might say to Descartes in jest, "You said that you knew your mind most clearly! But you've failed to complete your Copernican revolution."(Descartes 118)

Works Cited

Descartes, René, Gottfried Wilhelm. Leibniz, and Benedictus De. Spinoza. *The Rationalists. Discourse on Method. The Ethics. Meditations. Discourse on Metaphysics. The Monadology*. New York: Anchor, 1974. Print.

Kant, Immanuel, Paul Guyer, and Allen W. Wood. "Introduction and The Transcendental Aesthetic." *Critique of Pure Reason*. Cambridge: Cambridge UP, 1998. N. pag. Print.

Kant, Immanuel, Gary Hatfield, Karl Ameriks, and Desmond M. Clarke.*Immanuel Kant Prolegomena to Any Future Metaphysics*. Cambridge: Cambridge UP, 2004. Print.